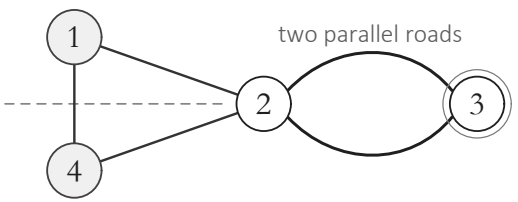
The fold keeps every count to 3

Each road costs one step. The folded arrows count choices from one starting station.

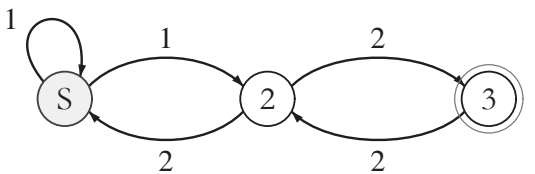
01 Two maps of the same walks

Original stations



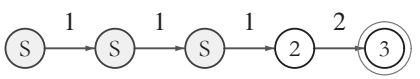
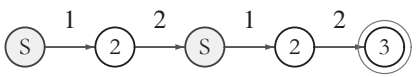
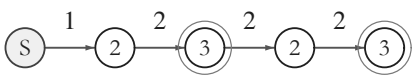
– – – Mirror line fixes 2 and 3

Folded states



S groups 1 and 4. Its loop costs one step.

02 Exactly four steps from 1 to 3

Original stations	*Folded route and choices by step*	*Product*	*Walks*
$1 \to 4 \to 1 \to 2 \to 3$		$1 \times 1 \times 1 \times 2$	2
$1 \to 2 \to 1 \to 2 \to 3$ $1 \to 2 \to 4 \to 2 \to 3$		$1 \times 2 \times 1 \times 2$	4
$1 \to 2 \to 3 \to 2 \to 3$		$1 \times 2 \times 2 \times 2$	8
		$2 + 4 + 8$	14

03 Why the count survives

The group map P

	S	2	3
1	1	0	0
2	0	1	0
3	0	0	1
4	1	0	0

Rows 1 and 4 map to S.

$AP = PB \Rightarrow A^nP = PB^n$

The mode that reverses sign

	1	2	3	4
u	+1	0	0	−1
Au	−1	0	0	+1

The pattern changes sign and stays zero at 3.

Its factor $1 + z$ cancels in the walk series.